

# MATHEMATICAL MODELING OF HORIZONTAL DISPLACEMENT OF ABOVE-GROUND GAS PIPELINES

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## Abstract

The modern geodetic equipment allows observations as soon as possible, providing high accuracy and productivity. Achieving high accuracy of measurement is impossible without taking into account external factors that create influence on an observation object. Therefore, in order to evaluate an influence of thermal displacement on the results of geodetic monitoring a mathematical model of horizontal displacement of above-ground pipelines was theoretically grounded and built. In this paper we used data of experimental studies on the existing pipelines “Soyuz” and “Urengoy-Pomary-Uzhgorod”. Above-ground pipeline was considered as a dynamic system “building-environment”. Based on the characteristics of dynamic systems the correlation between the factors of thermal influence and horizontal displacement of the pipeline axis was defined.

Establishing patterns between input factors and output response of the object can be useful not only for geodetic control, but also for their consideration in the design of new objects. It was investigated that the greatest influence on the accuracy of geodetic observations can create dispersion of high-frequency oscillations caused by daily thermal displacement. The magnitude of displacement exceeds actual measurement error.

The article presents the results of calculation of high-frequency oscillations of above-ground gas pipeline.

The result made it possible to substantiate the accuracy and methodology of geodetic observations of the horizontal displacement of pipeline axes taking into account an influence of cyclical thermal displacement.

Research results were recommended for use in practice for enterprises that serve the main gas pipelines and successfully tested by specialists of PJSC “Ukrtransgaz” (Kharkiv, Ukraine) during the technical state control of aerial pipeline crossing in Ukraine and also can be used to form the relevant regulations.

**Keywords:** above-ground pipeline, thermal displacement, dynamic model, geodetic control.

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## 1. Introduction

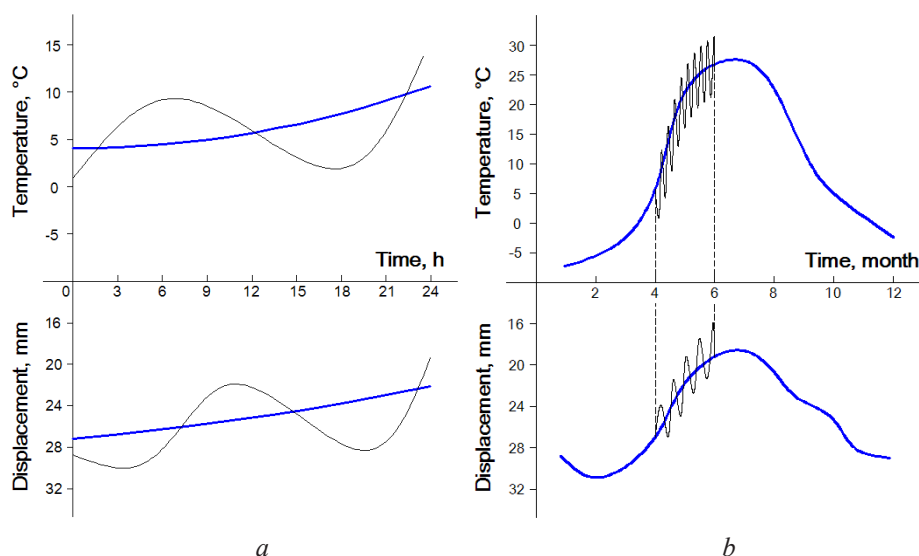
Among the most important scientific and technical problems of the XXI century [1] is the problem of evaluation of the technical condition and the continuation of safe operation of potentially dangerous objects. Among them there are main gas pipelines, most of which worked more than half of their life project. According to the “Rules of technical operation of main gas pipelines” [2], observation complex of above-ground pipelines (**Fig. 1**) includes the works to determine the spatial position of the pipeline axis by geodetic methods. Stress-strain state and evaluation of the bearing capacity of the pipeline are calculated on this basis.



**Fig. 1.** Aerial “Urengoy-Pomary-Uzhgorod” gas pipeline crossing above Bystrytsia Solotvynska River (Ukraine)

One of the main tasks of geodetic science and practice during the construction of buildings, installation of process equipment and control of the technical condition of the objects in the operation is to establish the required measurement accuracy [3, 4]. The required accuracy is the amount of acceptable error in determining displacement relating to stable point, which is considered as the original. To select the optimal methods of control of above-ground pipeline location and equipment for these works we must fulfill a priori accuracy calculation of geodetic observations.

The results of geodetic observations of deformations of any buildings integrate the patterns of complex interaction of “building-environment.” The establishment of such patterns may be useful not only for geodetic control, but also for their consideration in the design of new buildings. “Environment” subsystem includes short-time loads of non-load influence factors. These include: changes in ambient temperature, atmospheric and soil moisture, solar radiation. The processes of changing temperatures and horizontal pipeline displacement have a harmonious character. High-frequency daily oscillations and the main harmonic, which is seasonal, are occurred.



**Fig. 2.** Schematic representation of pipeline displacement under changing temperature conditions:  
*a* – daily harmonic; *b* – seasonal harmonic

The main influence on the accuracy of observations will create dispersion of high-frequency oscillations that can exceed actual measurement errors.

Ukrainian and foreign regulations [2, 5] lack any requirements for appointment of observational accuracy for planned and high-altitude displacements of above-ground pipelines. Therefore, the line-maintenance services, serving the main gas pipelines, require to determine the plan position of the pipeline axes with maximum accuracy. The problem of observation accuracy justification of high-altitude displacement is discussed in detail in [6].

The aim is to develop a mathematical model of horizontal displacement of above-ground pipeline axis to evaluate the influence of high-frequency oscillations on the results of geodetic observations. To achieve this aim it is necessary to solve the following problems:

1. To set the ratio between input factors (factors of thermal influence) and initial reaction of the building (horizontal displacement of the pipeline) based on the characteristics of dynamic systems.
2. To prove the methodology and accuracy of observation of horizontal displacements of above-ground pipelines in view of high-frequency oscillations caused by thermal influence factors.

## 2. Materials and methods of research

High accuracy of observations of the horizontal displacement of pipeline axes is impossible without the influence of cyclic thermal displacement. Depending on changes in air temperature, position of the pipeline axis, during the day, varies 10 mm or more [7]. The difference between

the surface temperature of the pipe on the sunny and shady sides creates additional tension in the pipeline [8, 9], the result of which is also a horizontal shift of the pipes.

The author's experimental researches of cyclic thermal displacement on operating pipelines "Soyuz" and "Urengoy-Pomary-Uzhgorod" confirm the effect of ambient temperature on the pipeline. Period of maximum amplitude of daily changes in air temperature was chosen. In the foothills of the Carpathian Mountains (Western Ukraine) this period is in June. During observations were recorded: air temperature  $t$ , the temperature difference of solar and shadow sides of the pipe  $\Delta T$  within the reference section and horizontal displacement  $S$ . Diameter of investigated area – 1420 mm. The temperature of the pipe was determined using a portable pyrometer with an accuracy within 0,1 °C. Within 13 hours there were 41 series of observations. The temperature was ranged from 13 to 37 °C.

Graphic results in [10] clearly trace the pattern of pipeline displacement during the observation period. The greatest value of thermal displacement of pipeline axis corresponds to the period of maximal air temperature, confirming the lack of inertial delay of building reaction.

A similar trend is observed in [11], where the largest displacement amplitude of the bridge corresponds to the period of maximum cooling of the metal from which it is made.

Based on the correlation analysis [12] it is found that 62 % of the daily thermal displacement of above-ground pipeline depends on the air temperature and the uneven side heat of the pipe by solar rays.

### 3. Development of a dynamic model of horizontal displacement of above-ground pipeline

#### 3. 1. Model development

The theory of dynamical systems, arising from automatic control theory, is successfully used in various fields of science and technology, using mathematical modeling techniques to solve the tasks. One of the advantages of mathematical models of dynamic systems is taking into account the inertial delay of the building when displaying natural properties of convert of the input data into the system reaction [13].

Let's consider a system of "building-environment" as a dynamic system. Previously it was found that the main factors affecting the horizontal displacement of the pipeline are air temperature and uneven side heat of the pipe by solar rays. These factors are input factors of the dynamic system. The responses of the system, or output variables, are pipeline displacement.

The dynamic model of deformation has three main components [14]. The first component reflects the dynamic properties of the system "building-environment"; the second – the fate of the movements that occur under the influence of the main input actions; the third component – includes displacement arising as a result of unaccounted factors (noise component).

Differential and recurrence equations establish a correspondence between the input and output variables. Let's introduce the designation of these processes:

$S_k$  – horizontal displacement of the pipeline;

$t_k$  – air temperature;

$\Delta T_k$  – pipe surface temperature gradient between sunny and shady sides;

$k$  – number of series of observations.

It is known that when inertial delay of the building response doesn't exceed 1/4 of the basic harmonic for modeling of the investigated process should be used the first order differential equations [13]. Above-ground pipeline is an open metal structure. Therefore, the processes of heat transfer in the system "building-environment" are no time delay.

Taking into account designation and the fact of the absence of inertial delay, let's write the model of horizontal displacement of the pipeline  $S_k$  in the recurrent form based on first order equations [13]

$$S_k = \varphi S_{k-1} + \beta_1 t_k + \beta_2 \Delta T_k, \quad (1)$$

where  $\varphi$  – dynamic factor,  $\beta_1$ ,  $\beta_2$  – factors reflecting the impact of input factors on pipeline displacement.

Initial data for model development:

- the results of observations of the horizontal displacement of the reference section  $S_k$  of aerial “Urengoy-Pomary-Uzhgorod” gas pipeline crossing (pipe diameter 1420 mm) made using geodetic methods;
- the results of measurements of air temperature  $t_k$ ;
- the results of determination of the temperature difference between the surface of the pipeline on the sunny and shady sides within the reference section.

Methods of observations and the results are detailed written in [12]. Reference section was selected for modeling, because values of its displacement had the largest amplitude. The observations were carried out in the period of maximum change in temperature during the day (June).

An evaluation of model parameters describing displacement of one point (reference section of the pipeline) should start with the expectation approximation. Conditional expectation of equation (1) is:

$$M\{S_k / S_{k-1}, t_k, \Delta T_k\} = \hat{S}_{k/k-1} = \hat{\varphi}S_{k-1} + \hat{\beta}_1 t_k + \hat{\beta}_2 \Delta T_k. \quad (2)$$

Model development is beginning to evaluate the parameters  $\varphi, \beta_1, \beta_2$  on the basis of daily observations of the input  $t_k, \Delta T_k$  and output  $S_k$ . To do this, let's find the minimum of the function [15]

$$F(\varphi, \beta_1, \beta_2) = \sum_{k=2}^N (S_k - \hat{S}_{k/k-1})^2, \quad (3)$$

$$\left. \begin{aligned} \frac{\partial F}{\partial \varphi} &= -2 \sum_{k=2}^N (S_k - \hat{\varphi}S_{k-1} - \hat{\beta}_1 t_k - \hat{\beta}_2 \Delta T_k) \cdot S_{k-1} = 0, \\ \frac{\partial F}{\partial \beta_1} &= -2 \sum_{k=2}^N (S_k - \hat{\varphi}S_{k-1} - \hat{\beta}_1 t_k - \hat{\beta}_2 \Delta T_k) \cdot t_k = 0, \\ \frac{\partial F}{\partial \beta_2} &= -2 \sum_{k=2}^N (S_k - \hat{\varphi}S_{k-1} - \hat{\beta}_1 t_k - \hat{\beta}_2 \Delta T_k) \cdot \Delta T_k = 0, \end{aligned} \right\} \quad (4)$$

This gives the system of normal equations to calculate parameters  $\varphi, \beta_1, \beta_2$ :

$$\left\{ \begin{aligned} \hat{\varphi} \sum_{k=2}^N S_{k-1}^2 + \hat{\beta}_1 \sum_{k=1}^N S_{k-1} t_k + \hat{\beta}_2 \sum_{k=1}^N S_{k-1} \Delta T_k - \sum_{k=1}^N S_k S_{k-1} &= 0, \\ \hat{\varphi} \sum_{k=2}^N S_{k-1} t_k + \hat{\beta}_1 \sum_{k=1}^N t_k^2 + \hat{\beta}_2 \sum_{k=1}^N t_k \Delta T_k - \sum_{k=1}^N S_k t_k &= 0, \\ \hat{\varphi} \sum_{k=2}^N S_{k-1} \Delta T_k + \hat{\beta}_1 \sum_{k=1}^N t_k \Delta T_k + \hat{\beta}_2 \sum_{k=1}^N \Delta T_k^2 - \sum_{k=1}^N S_k \Delta T_k &= 0. \end{aligned} \right. \quad (5)$$

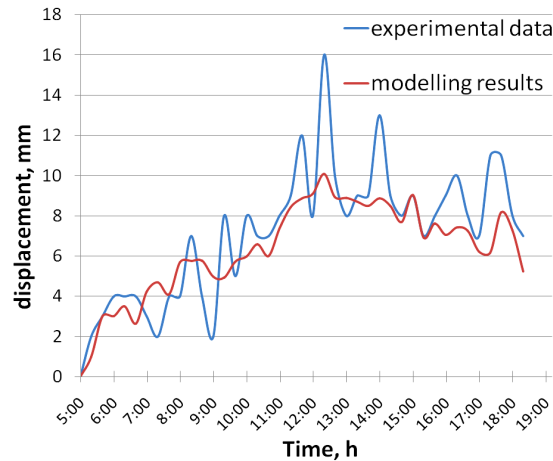
Evaluation of unknown parameters in this system of normal equations should perform according to centered input and output values. This does not alter the structure of the system of normal equations and replace  $S_k, t_k, \Delta T_k$  by their centered values:

$$\overset{\circ}{S}_k = S_k - \bar{S}, \quad \overset{\circ}{t}_k = t_k - \bar{t}, \quad \overset{\circ}{\Delta T}_k = \Delta T_k - \bar{\Delta T}, \quad (6)$$

where  $\bar{S}, \bar{t}, \bar{\Delta T}$  – arithmetic average. Solving the system of equations, and substituting the unknown parameters in (1), we obtain short-period nature model of horizontal displacement process of the pipeline:

$$S_k = 0.04S_{k-1} + 0.38t_k + 0.15\Delta T_k. \quad (7)$$

The graphs of pipeline axis displacements derived from empirical data and modeling results are presented in **Fig. 3**.



**Fig. 3.** The results of modeling short-period horizontal displacement of above-ground pipeline

After completing the accuracy evaluation of the calculated parameters, the following values are obtained:  $m_\varphi = 0.01$ ;  $m_{\beta_1} = 0.01$ ;  $m_{\beta_2} = 0.02$ .

### 3. 2. Evaluation of the influence of high-frequency oscillations on the accuracy of the horizontal displacement of the pipeline

In order to correct organization of geodetic observations we must ensure the conditions:

$$m_{xy0} \leq m_{xy}, \quad (8)$$

where  $m_{xy}$  – mean square error of geodetic observations of the planned position of the pipeline axis;  $m_{xy0}$  – mean square error, which describes the range of possible high-frequency oscillations.

In [13], based on the tools of frequency characteristics of dynamic systems, an equation is obtained that establishes a connection between the average square error of input and output for dynamic systems. Let's write the equation due to our case (two input parameters):

$$m_{xy0} = \sqrt{\frac{\beta_1^2 m_t^2 + \beta_2^2 m_{\Delta T}^2}{1 + 2\varphi \cos \omega\tau + \varphi^2}}, \quad (9)$$

where the parameters  $\varphi$ ,  $\beta_1$ ,  $\beta_2$  are obtained according to modeling results (7);  $m_t$ ,  $m_{\Delta T}$  – mean square errors of amplitude changes in air temperature and surface temperature gradient of the pipe;

$\omega\tau = \frac{\pi}{12} \cdot 0.3 = 4.5^\circ$  because a half of high-frequency daily fluctuation harmonic is used for prediction (12 hours of observations); sampling interval is 20 minutes (0.333 hours) – time between observation cycles.

Errors  $m_t$ ,  $m_{\Delta T}$  are calculated using the formulas

$$m_t = \sqrt{\sigma_t^2 + m_{tcalc}^2}, \quad m_{\Delta T} = \sqrt{\sigma_{\Delta T}^2 + m_{\Delta Tcalc}^2}, \quad (10)$$

where  $\sigma_t^2$ ,  $\sigma_{\Delta T}^2$  – dispersions of temperature fluctuations;  $m_{tcalc} = \pm 1^\circ\text{C}$  – measurement error of air temperature;  $m_{\Delta Tcalc} = \pm 0.1^\circ\text{C}$  – measurement error of surface temperature of the pipe using a portable pyrometer

Substituting in the formula (9) the parameters of the model (7), and the calculated errors  $m_t$  and  $m_{\Delta T}$ , as a result, we get:

$$m_{xy0} = \sqrt{\frac{\beta_1^2 m_t^2 + \beta_2^2 m_{\Delta T}^2}{1 + 2\varphi \cos \omega\tau + \varphi^2}} = \sqrt{\frac{0.38^2 \cdot 42.5 + 0.15^2 \cdot 1.11}{1 + 2 \cdot 0.04 \cdot \cos 4.5^\circ + 0.04^2}} = \sqrt{\frac{6.16}{1.081}} = 2.4 \text{ mm}. \quad (11)$$

This result indicates that the horizontal displacement of the pipeline axis, the value of which is determined in the geodetic control, is exposed to high-frequency temperature fluctuations.

Where there is a need to assign the highest possible accuracy of geodetic observations according to planned displacement of the pipeline axis, which is characterized by the mean square error  $m_{xy}$ , enforcement of conditions  $m_{xy} > m_{xy0} = 2.4$  mm will allow neglect dispersion of high-frequency oscillations, which are caused by temperature deformations.

#### 4. Discussion of research results

Beam above-ground gas pipelines (**Fig. 1**) are the most common in Ukraine. Therefore, experimental research was carried out on buildings of this type. The results are typical for aerial crossing of 1420 mm beam above-ground gas pipelines.

We believe that the correct method during the geodetic control of the position of the pipeline axis is to carry observations in the same periods of the day. High-frequency oscillations will be fixed at the same level of their amplitude, or in one of the culminations (period of maximum cooling or heating of the pipes).

However, it can choose the method of random selection of observation points in relation to high-frequency oscillations. It must ensure the conditions (8) – observation accuracy of horizontal displacement of above-ground pipeline will not exceed the mean square error of high-frequency oscillations. Therefore, it is recommended that the observation accuracy of horizontal displacement of above-ground pipeline axes is not higher than 2.4 mm.

Appointment of accuracy on this principle can be performed only if the value of the error  $m_{xy} = 2.4$  mm does not create a significant influence on the evaluation of the object.

It should be noted that the value of the mean square error of high-frequency oscillations of above-ground gas pipelines is not constant. It will vary depending on the air temperature conditions for certain areas, and the amount of solar radiation, which depends on the geographical latitude.

#### 5. Conclusions

1. Selection justification of mathematical model of horizontal displacement of above-ground pipeline axis is done.

2. Value  $m_{xy0} = 2.4$  mm, which describes the range of possible high-frequency oscillations of beam above-ground pipelines ( $D = 1420$  mm) caused by daily thermal displacement, is calculated.

3. An accuracy of geodetic observations of the horizontal displacement of the pipeline axes is proved taking into account the impact of cyclical thermal displacement.

4. Research results are recommended for use in practice for enterprises that serve the main gas pipelines and successfully tested during the technical state control of aerial pipeline crossing in Ukraine

We believe that further studies related to the implementation and development of automated systems for monitoring oil and gas facilities are promising.

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